Physically based parameterizations of the short-wave radiative characteristics of weakly absorbing optically thick media: application to liquid-water clouds

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We propose the physically based parameterization of the radiative characteristics of liquid-water clouds as functions of the wavelength, effective radius, and refractive index of particles, liquid-water path, ground albedo, and solar and observation angles. The formulas obtained are based on the approximate analytical solutions of the radiative transfer equation for optically thick, weakly absorbing layers and the geometrical optics approximation for local optical characteristics of cloud media. The accuracy of the approximate formulas was studied with an exact radiative transfer code. The relative error of the approximate formula for the reflection function at nadir observations was less then 15% for an optical thickness larger than 10 and a single-scattering albedo larger than 0.95. © 1998 Optical Society of America

OCIS codes: 010.1290, 260.2160, 280.1310, 290.1090, 290.1310, 290.4210.

1. Introduction

Clouds are strong modulators of the short-wave and long-wave components of the Earth's radiation budget. Their optical and microphysical properties were studied in numerous books and papers.[1–11] A number of approximate[2–4,10,12,13] and exact[12,13] methods to calculate radiative characteristics of water clouds were developed. One of the most powerful methods of calculation of the radiative characteristics of water clouds is based on the asymptotic formulas that can be derived from the radiative transfer equation at large values of the optical thickness τ.[2,4,10–16] Within the framework of this asymptotic theory for optically thick layers the reflection $R(\mu, \mu_0, \phi)$ and the transmission $T(\mu, \mu_0, \phi)$ functions of a plane-parallel light-scattering layer can be written in the following form (see Appendix B for a list of symbols used in this paper)[2–4,10–16]:

$$R(\mu, \mu_0, \phi) = R_\infty(\mu, \mu_0, \phi) - \frac{ml\,\exp(-2k\tau)K(\mu)K(\mu_0)}{1 - l^2\exp(-2k\tau)}, \tag{1}$$

$$T(\mu, \mu_0) = \frac{m\,\exp(-k\tau)K(\mu)K(\mu_0)}{1 - l^2\exp(-2k\tau)}, \tag{2}$$

$$l = 2\int_0^1 K(\mu)P(-\mu)\mu\mathrm{d}\mu,$$

$$m = 2\int_{-1}^1 P^2(\mu)\mu\mathrm{d}\mu. \tag{3}$$

In Eqs. (1)–(3), μ_0 is the cosine of the solar zenith angle ϑ_0, μ is the cosine of the emerging zenith angle ϑ, ϕ is the azimuth angle measured from the solar plane, k is the diffusion exponent, $K(\mu)$ is the escape function, $P(\mu)$ is the diffusion pattern, l and m are scalar constants determined by the local optical properties of the medium, and $R_\infty(\mu, \mu_0, \phi)$ is the reflection function of a semi-infinite layer that has the same local optical properties as the finite layer under investigation. As follows from Eq. (2), the radiance transmitted by an optically thick layer does not depend on the azimuth ϕ. Equations (1) and (2) are

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Received 17 October 1997; revised manuscript received 18 February 1998.
0003-6935/98/214750-08$15.00/0

accurate to within 1% for $(1 - g)\tau \geq 1.45$,[11] where $g = 1/2 \int_0^\pi p(\theta)\sin\theta \cos\theta \, d\theta$ is the asymmetry parameter, $p(\theta)$ is the phase function, and θ is the scattering angle. The value of g is approximately 0.84 for liquid-water clouds in the midvisible region, where the single-scattering albedo $\omega_0 \sim 1$. Thus Eqs. (1) and (2) are applicable for calculations of radiative characteristics of water clouds of optical thickness $\tau \geq 9$ with an accuracy within 1%. Note that most of water clouds have an optical thickness in the range 7–70.[11] Thus Eqs. (1) and (2) can be applied to modeling radiative properties of real-world clouds. Moreover, thin clouds are usually inhomogeneous in all directions, and one cannot apply the radiative transfer equation for horizontally uniform plane-parallel light scattering media in this case.

To use Eqs. (1) and (2) one should calculate constants k, l, m and functions $K(\mu)$ and $R_\infty(\mu, \mu_0, \phi)$. Different numerical methods for making such computations were discussed by Nakajima and King.[14] They also proposed efficient numerical algorithms for computing the asymptotical functions and constants that are valid for any single-scattering albedos. However, our own point of view is that, with the advent of fast computers and efficient multiple-scattering codes, the main usefulness of approximations (1) and (2) lies in their ability to be manipulated analytically. With this in mind, it is important to find approximate analytical equations for the values of k, l, and m and functions $K(\mu)$ and $R_\infty(\mu, \mu_0, \phi)$. Analytical approximations for the constants k, l, and m that are applicable over the full range of single-scattering albedos were obtained by King[15] and by King and Harshvardhan.[16] Simple parameterizations for functions $K(\mu)$ and $R_\infty(\mu, \mu_0, \phi)$ at any value of ω_0 have not been proposed. Thus we intend in this paper to replace Eqs. (1) and (2) with simpler ones that can be analytically manipulated. To derive these new equations we assume that the probability of photon absorption by particles $\beta = 1 - \omega_0$ is small (≤ 0.05). Such an assumption holds for many natural media, including clouds, ocean whitecaps, snow, and ice fields in visible and near-infrared (NIR) regions of the solar spectrum. For example, the value of β for liquid-water clouds is less then 5% at the wavelength $\lambda \leq 2.2$ μm.[17]

2. Modified Asymptotic Theory for Weakly Absorbing Media

Let us suppose now that the probability of photon absorption β and the diffusion exponent[2,13] $k = [3\beta(1 - \omega_0 g)]^{1/2} + o(\beta)$ are small. In this case the following power-series expansions in k for the constants and functions that appear in Eqs. (1) and (2) can be obtained[2,13]:

$$m = \frac{8k}{3(1-g)} + o(k^3), \quad l = 1 - 2q_0k + 2{q_0}^2k^2 + o(k^3), \tag{4}$$

$$K(\mu) = K_0(\mu)(1 - q_0k) + o(k^2), \tag{5}$$

$$R(\mu, \mu_0, \phi) = {R_\infty}^0(\mu, \mu_0, \phi) - \frac{4kK_0(\mu)K_0(\mu_0)}{3(1-g)} + o(k^2), \tag{6}$$

where $q_0 = [2/(1 - g)] \int_0^1 K_0(\mu)\mu^2 d\mu$, $o(k^n)$ denotes the terms of order k^n or higher, and $K_0(\mu)$ and ${R_\infty}^0(\mu, \mu_0, \phi)$ denote the escape function and the reflection function, respectively, of a semi-infinite nonabsorbing atmosphere. To apply Eqs. (1)–(6) one should calculate functions $K_0(\mu)$ and ${R_\infty}^0(\mu, \mu_0, \phi)$. These functions depend on the phase function only. This dependence for the escape function of the nonabsorbing medium is rather weak and almost linear[4,13,18–20]:

$$K_0(\mu) = a + b\mu, \tag{7}$$

where values a and b are constants. These constants can be obtained from formulas for the first and the second moments of the escape function[13]:

$$2\int_0^1 K_0(\mu)\mu d\mu = 1, \tag{8}$$

$$2\int_0^1 K_0(\mu)\mu^2 d\mu = C, \tag{9}$$

where $C = (1 - g)q_0 \approx 5/7$ for water clouds [see Eq. (23) of Ref. 15]. From Eqs. (7)–(9) it follows that

$$a = 9 - 12C, \qquad b = 18C - 12. \tag{10}$$

Thus for the escape function [Eqs. (7) and (10)] one can obtain

$$K_0(\mu) = 3(1 + 2\mu)/7 \tag{11}$$

at $C = 5/7$. The error in this simple formula is less then 5% at $\mu \geq 0.25$. Note that Eq. (11) satisfies the normalization condition [Eq. (8)]. This is an important point.

The reflection function ${R_\infty}^0(\mu, \mu_0, \phi)$ depends on the phase function. The approximate formula for this function in the special case of liquid-water clouds is presented in Appendix A. A more-general result can be obtained for the value of ${R_\infty}^0(\mu, \mu_0, \phi)$ averaged on the the azimuth. The value of

$${R_\infty}^0(\mu, \mu_0) = \frac{1}{2\pi}\int_0^{2\pi} {R_\infty}^0(\mu, \mu_0, \phi)d\phi \tag{12}$$

depends only slightly on the type of the phase function and can be approximated by the functional form, which approximately holds for the case of isotropic scattering:

$${R_\infty}^0(\mu, \mu_0) = \varepsilon_0 \frac{1 + \varepsilon_1\mu\mu_0 + \varepsilon_2(\mu + \mu_0)}{\mu + \mu_0}, \tag{13}$$

where ε_0, ε_1, and ε_2 are constants. Indeed, for isotropic scattering it follows that[18]

$$R_\infty{}^0(\mu, \mu_0) = \frac{H(\mu)H(\mu_0)}{4(\mu + \mu_0)}. \tag{14}$$

The behavior of the function $H(\mu)$ is almost linear: $H(\mu) = a_1 + a_2\mu$, where it follows approximately that $a_1 = 1$ and $a_2 = 2$ (see Table XI of Ref. 18). The functional form [Eq. (13)] follows from Eq. (14) if one assumes a linear dependence of the function H on μ. Constants ε_0, ε_1, and ε_2 were found by Zege *et al.*[14] ($\varepsilon_0 = 0.49$, $\varepsilon_1 = 4$, $\varepsilon_2 = 0$) by comparison of Eq. (13) with the results of numerical calculations of $R_\infty{}^0$ (μ, μ_0) for liquid-water clouds. The error in Eq. (13) is less then 10% at $\mu, \mu_0 \geq 0.2$.[4] This equation is especially useful at nadir measurements. In this case the reflection function does not depend on the azimuth at all.

Unfortunately, Eqs. (1), (2), (4)–(6), (11), and (13) can be applied only at $\beta \leq 0.005$.[2] This value for water clouds in the NIR can be ten times larger.[17] Thus it is necessary to generalize this approach to larger values of β. To do so we note that it follows from Eqs. (4) and (5) that

$$mK(\mu)K(\mu_0) \approx mK_0(\mu)K_0(\mu_0), \tag{15}$$

and Eqs. (1) and (2) can be written in the following approximate form:

$$R(\mu, \mu_0, \phi) = R_\infty(\mu, \mu_0, \phi) - T(\mu, \mu_0)l \exp(-k\tau), \tag{16}$$

$$T(\mu, \mu_0) = \frac{m \exp(-k\tau)K_0(\mu)K_0(\mu_0)}{1 - l^2 \exp(-2k\tau)}. \tag{17}$$

Constants l and m in Eqs. (16) and (17) can be found by comparison of the spherical albedo

$$R = \frac{2}{\pi}\int_0^{2\pi} d\phi \int_0^1 \mu d\mu \int_0^1 \mu_0 d\mu_0 R(\mu, \mu_0, \phi) \tag{18}$$

and the global transmittance

$$T = 4\int_0^1 \mu d\mu \int_0^1 \mu_0 d\mu_0 T(\mu, \mu_0) \tag{19}$$

obtained from Eqs. (16)–(19) with the analytical results derived by Rozenberg at $\beta \leq 0.05$ (Ref. 10):

$$R = \frac{\sinh(x)}{\sinh(x + y)}, \qquad T = \frac{\sinh(y)}{\sinh(x + y)}, \tag{20}$$

where $x = k\tau$ and $y = 4k/[3(1 - g)]$.

It follows from Eqs. (16)–(19) and (8) that

$$R = R_\infty - Tl \exp(-k\tau), \tag{21}$$

$$T = \frac{m \exp(-k\tau)}{1 - l^2 \exp(-2k\tau)}. \tag{22}$$

Thus one can find by comparison of Eqs. (20)–(22) that $m = 1 - l^2$ and $l = R_\infty = \exp(-y)$. Note from these results that it follows approximately for thick weakly absorbing layers [Eqs. (3)] that $R_\infty \approx 2 \int_0^1 K(\mu)P(-\mu)\mu d\mu \approx [1 - 2 \int_{-1}^1 P^2(\mu)\mu d\mu]^{1/2}$. Finally, the values of $R(\mu, \mu_0, \phi)$ and $T(\mu, \mu_0)$ have the following form:

$$R(\mu, \mu_0, \phi) = R_\infty(\mu, \mu_0, \phi) - T(\mu, \mu_0)\exp(-y - x), \tag{23}$$

$$T(\mu, \mu_0) = \frac{\sinh(y)}{\sinh(x + y)} K_0(\mu)K_0(\mu_0), \tag{24}$$

where the reflection function $R_\infty(\mu, \mu_0, \phi)$ can be expressed through the value of the reflection function of a semi-infinite nonabsorbing medium $R_\infty{}^0(\mu, \mu_0, \phi)$ (Refs. 4 and 10):

$$R_\infty(\mu, \mu_0, \phi) = R_\infty{}^0(\mu, \mu_0, \phi)\exp\left[-y \frac{K_0(\mu)K_0(\mu_0)}{R_\infty{}^0(\mu, \mu_0, \phi)}\right]. \tag{25}$$

Note that at small values of β the value of y is small as well, and one can obtain Eq. (6) from Eq. (25).

Equations (23) and (24) are simpler than Eqs. (1) and (2). They can be used [see Eq. (8)] for calculations of the plane albedo

$$R(\mu_0) = \frac{1}{\pi}\int_0^{2\pi} d\phi \int_0^1 \mu d\mu R(\mu, \mu_0, \phi), \tag{26}$$

the diffused transmittance

$$T(\mu_0) = 2\int_0^1 \mu d\mu T(\mu, \mu_0), \tag{27}$$

and the absorptance $A = 1 - R - T$.

The results are presented in Appendix C, where we used the parameters y and $z = x/y$ instead of y and x proposed by Rozenberg[10] [Eq. (20)]. The parameter y at $\omega_0 \approx 1$ is proportional to the similarity parameter[13] $s = [(1 - \omega_0)/(1 - g\omega_0)]^{1/2}$ introduced by van de Hulst,[13] and the value of z is proportional to the scaled optical thickness[13] $\tau = (1 - g)\sigma_{ext}L$, where σ_{ext} is the extinction coefficient and L is the geometrical thickness. Accounting for the Lambertian underlying surface with the albedo r_s is simple[11]:

$$R_*(\mu, \mu_0, \phi) = R(\mu, \mu_0, \phi) + \frac{r_s T(\mu)T(\mu_0)}{1 - Rr_s}, \tag{28}$$

$$T_*(\mu, \mu_0) = T(\mu, \mu_0) + \frac{r_s R(\mu)T(\mu_0)}{1 - Rr_s}, \tag{29}$$

where $R_*(\mu, \mu_0, \phi)$ and $T_*(\mu, \mu_0)$ denote, respectively, the reflection function and the transmittance function of a medium with an underlying surface. In Eqs. (28) and (29) it is assumed that there is no absorption in the medium beneath the scattering layer, and these equations can also be in error for large surface albedos.[21]

Thus we have presented simple equations for calculating the radiative transfer characteristics of

weakly absorbing optically thick media (see Appendix C). The user of these equations does not need to create special codes to calculate auxiliary constants and functions, as was necessary for the standard theory for optically thick layers [Eqs. (1) and (2)].

3. Application to Liquid-Water Clouds

It follows from Eqs. (23), (24), (28), and (29) that the radiative characteristics of weakly absorbing media depend mostly on the geometry, the surface albedo, and two local optical parameters of a scattering layer:

$$y = 4\left[\frac{\beta}{3(1-g)}\right]^{1/2}, \qquad z = 3/4(1-g)\sigma_{\text{ext}}L. \quad (30)$$

The formulas presented in Appendix C can be applied to studies of light propagation through various light-scattering media. Let us consider now a special case of liquid-water clouds at short wavelengths. The average size of water drops (4–20 μm) is larger then the wavelength of the incident radiation in the visible and NIR bands of the solar spectrum, and the absorption of photons by the drops is extremely low ($\beta \le 0.05$). Thus not only the radiative characteristics but also the local optical properties of water clouds can be calculated with simple approximate equations. One can find the local optical characteristics of water clouds from Eqs. (30) within the framework of the modified geometrical optics approximation[22] ($\lambda \ll a$, $4\pi a|m-1|/\lambda \gg 1$, where a is the radius of the drops and $m = n - i\chi$ is their refractive index):

$$\sigma_{\text{ext}} = \frac{1.5C_v}{a_{\text{ef}}}\left(1 + \frac{1.1}{x_{\text{ef}}^{2/3}}\right), \quad (31)$$

$$\beta = \sigma_{\text{abs}}/\sigma_{\text{ext}}, \qquad \sigma_{\text{abs}} = 1.2\alpha C_v(1+s)(1-\alpha a_{\text{ef}}), \quad (32)$$

$$1 - g = 0.118 + \frac{0.5 + 0.2c}{x_{\text{ef}}^{2/3}} - 0.75c\left(0.1 + \frac{1}{2x_{\text{ef}}^{2/3}}\right), \quad (33)$$

where $x_{\text{ef}} = 2\pi a_{\text{ef}}/\lambda$, $\alpha = 4\pi\chi/\lambda$, $c = 2\alpha a_{\text{ef}}$, $s = 0.34[1 - \exp(-8\lambda/a_{\text{ef}})]$, C_v is the volumetric concentration of particles, σ_{abs} is the absorption coefficient, and $a_{\text{ef}} = \langle a^3\rangle/\langle a^2\rangle$ is the effective radius of the particles. Angle brackets mean averaging on the droplet-size distribution. The relative error of Eqs. (31)–(33) in comparison with that derived from Mie theory is less than 10% for $\lambda \le 2.2$ μm, and values of a_{ef} range from 4 to 20 μm.[22] To make comparisons with Mie theory we used the gamma and log-normal particle-size distrubutions with different half-widths.

From Eqs. (30)–(33) one can obtain the following simple approximate formulas:

$$y = 6\sqrt{\alpha a_{\text{ef}}}, \qquad z = \frac{0.12w}{\rho a_{\text{ef}}}\left(1 + \frac{6}{x_{\text{ef}}^{2/3}}\right), \quad (34)$$

where $w = C_v\rho L$ is the liquid-water path (LWP) and $\rho = 1000$ kg/m^3 is the density of water. Therefore the radiative characteristics of water clouds at $\lambda \le 2.2$ μm depend mostly on two microstructure parameters: the effective radius of water drops a_{ef} and the LWP. The dependence of the shape of the drop-size distribution is weak. This result can be obtained from the Mie theory calculations as well.[6]

The simple solutions in Appendix C along with Eqs. (34) can be used to solve inverse cloud optics problems[23] and to investigate the dependence of the radiative characteristics of water clouds on their LWP, microstructure parameters, wavelength, and geometry. It is possible to use these solutions to parameterize the radiative properties of other weakly absorbing thick media (snow, foam, ice fields) in terms of microstructure parameters.

4. Accuracy of Approximate Solutions

Detailed studies of the accuracy of the approximate formulas presented here will be done elsewhere. In this paper we investigate the error of the approximate formula for the reflection function at nadir measurements [see Eqs. (23), (11), (13), (25), (28), and (34) and Appendix C):

$$R_*(1, \mu_0) = R(1, \mu_0) + \frac{r_s T^2 K_0(1)K_0(\mu_0)}{1 - r_s R}, \quad (35)$$

where

$$R(1, \mu_0) = R_\infty{}^0(1, \mu_0)\exp\left[-y\frac{K_0(1)K_0(\mu_0)}{R_\infty{}^0(1, \mu_0, \phi)}\right] - TK_0(1)K_0(\mu_0)\exp[-y(1+z)], \quad (36)$$

$$T = \frac{\sinh(y)}{\sinh[(1+z)y)]},$$

$$K_0(\mu_0) = 3/7(1 + 2\mu_0),$$

$$R_\infty{}^0(1, \mu_0) = 0.49\,\frac{1 + 4\mu_0}{1 + \mu_0}, \quad (37)$$

$$y = 6\sqrt{\alpha a_{\text{ef}}}, \qquad z = \frac{0.12w}{\rho a_{\text{ef}}}\left(1 + \frac{6}{x_{\text{ef}}^{2/3}}\right). \quad (38)$$

The accuracy of Eqs. (35)–(38) was studied with the exact radiative transfer code,[8] based on the discrete ordinate method of solving the radiative transfer equation.[14] The phase function and the single-scattering albedo in the exact code were calculated with Mie theory at the gamma particle-size distribution (see the Cloud C1 model in Ref. 1). The value of the effective radius of drops changed from 4 to 16 μm, the wavelength changed from 0.5 to 2.5 μm, the solar angle changed from 30 to 60 deg, the ground albedo changed from 0.1 to 0.5, and the LWP changed from 50 to 300 g/m^2, which are representative values for water clouds.[21]

The results of these computations are presented in Figs. 1–4. The optical thickness in Fig. 2 for several LWP's is in the range $\tau = 10$–70. The maximum

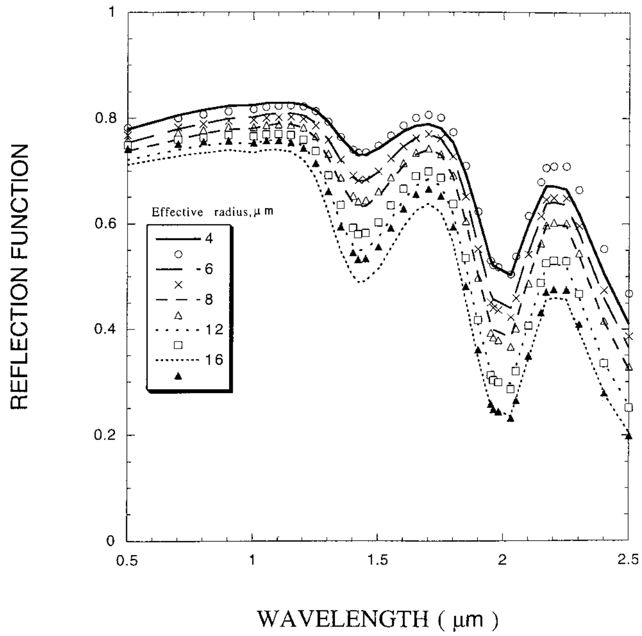

Fig. 1. Reflection function of water clouds at $\tau = 30$, $\mu = 1$, $\mu_0 = \sqrt{2}/2$, $r_s = 0$, and several values of the effective radii of droplets obtained by the exact radiative transfer code (curves) and approximate [Eqs. (35)–(38)] formulas.

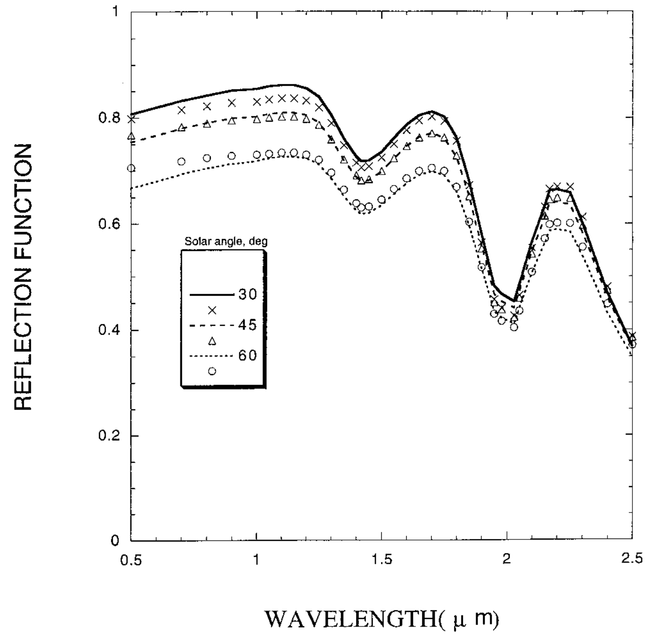

Fig. 3. Reflection function of water clouds at $\tau = 30$, $\mu = 1$, $a_{ef} = 6$ μm, $r_s = 0$, and several values of the solar zenith angle obtained by the exact radiative transfer code (curves) and approximate [Eqs. (35)–(38)] formulas.

probability of photon absorption in our calculations was 5%. One can see that the accuracy of the formulas obtained is rather good. Even at comparatively large absorption ($R = 0.25$; see Fig. 1 at $\lambda = 2$ μm) the accuracy is high. The accuracy is worse for thinner clouds (Fig. 2), as should be assumed. It is interesting to see that the error of Eq. (36) has a minimum at intermediate (and the most frequently occurring) values of the LWP (Fig. 2). This result stems from the fact that the modified asymptotic theory proposed here underestimates reflection functions of thin water clouds and overestimates the

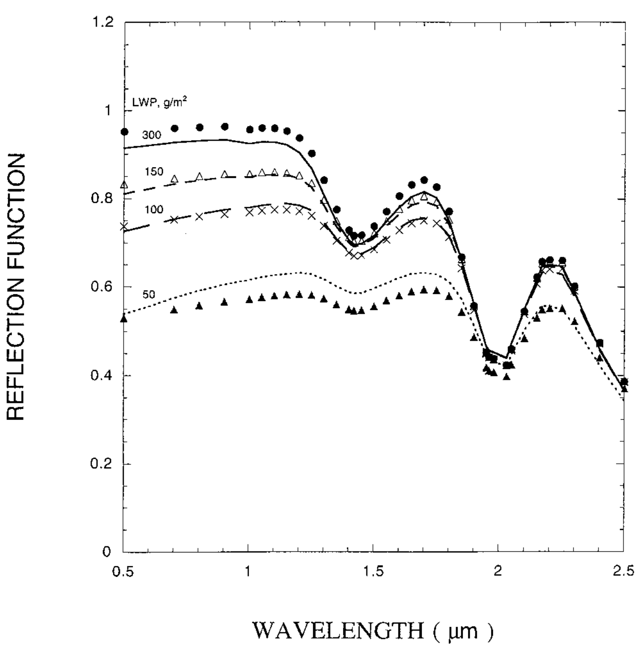

Fig. 2. Reflection function of water clouds at $a_{ef} = 6$ μm, $\mu = 1$, $\mu_0 = \sqrt{2}/2$, $r_s = 0$, and several values of the liquid-water path obtained by the exact radiative transfer code (curves) and approximate [Eqs. (35)–(38)] formulas.

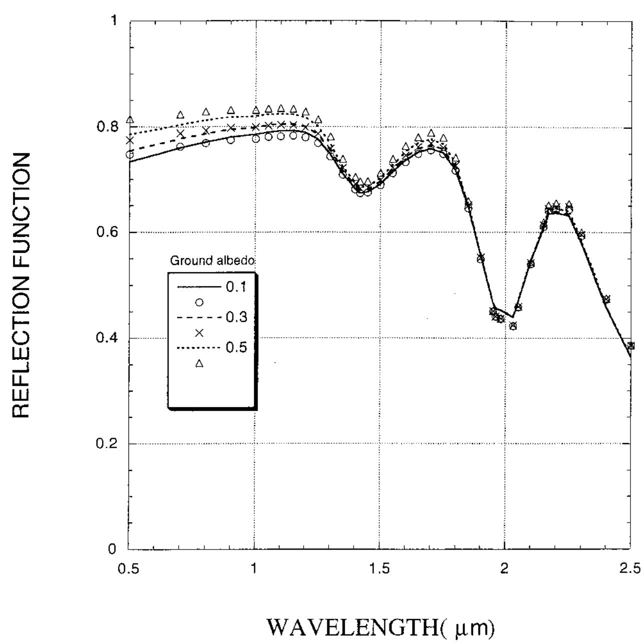

Fig. 4. Reflection function of water clouds at $\tau = 30$, $\mu = 1$, $\mu_0 = \sqrt{2}/2$, $a_{ef} = 6$ μm, and several values of the ground albedo obtained by the exact radiative transfer code (curves) and approximate [Eqs. (35)–(38)] formulas.

values of reflection functions of semi-infinite light-scattering layers.

The accuracy of the approximations is high at a solar angle of 45 deg (Fig. 3), but it decreases for smaller and larger solar angles. Moreover, our formulas (35)–(37) cannot be applied at $\vartheta_0 \approx 0, \pi/2$, because of the increase of the single light-scattering contribution at these angles. The accuracy decreases with the ground albedo r_s as well (Fig. 4). However, the influence of the ground albedo on the cloud reflection function at $\tau > 30$ is important only at $\lambda \leq 1.3\ \mu\text{m}$ (Fig. 4).

From analyses of Figs. 1–4 we come to the conclusion that we can use Eqs. (35)–(38) to estimate the reflection function $R(1, \mu_0)$ of water clouds with an error of less then 15% in the visible and NIR regions of the solar spectrum at $\tau \geq 10$ and $\omega_0 > 0.95$.

5. Conclusions

We have proposed here a modified asymptotic theory for the radiative characteristics of weakly absorbing optically thick media. The derived equations are simple (see Appendix C) and can be used as a powerful tool for estimation of the radiative properties of water clouds. It is possible to apply them to other weakly absorbing optically thick media, including snow and oceanic whitecaps. They are especially useful for solutions of inverse problems.[23]

We have demonstrated the accuracy of the modified asymptotic theory by using the exact radiative transfer code[8] and have found that the formula for the value of $R(1, \mu_0)$ is accurate within 15% for most representative values of the effective radius of water droplets ($4\ \mu\text{m} \leq a_{\text{ef}} \leq 20\ \mu\text{m}$), the liquid-water path ($50\ \text{g/m}^2 \leq w \leq 300\ \text{g/m}^2$), and the ground albedo ($0.1 \leq r_s \leq 0.5$).

Appendix A. Reflection of Light from Semi-Infinite Nonabsorbing Clouds

To derive the analytical expression for the reflection function of nonabsorbing semi-infinite clouds we divide the reflection function into two parts:

$$R_\infty{}^0(\mu, \mu_0, \phi) = R_{\text{ss}} + R_{\text{ms}}, \tag{A1}$$

where[18]

$$R_{\text{ss}} = \frac{p(\theta)}{4(\mu + \mu_0)} \tag{A2}$$

is the contribution of the single scattering to the value of $R_\infty{}^0(\mu, \mu_0, \phi)$ and R_{ms} represents photons that were scattered two or more times. The multiple light scattering in conservative semi-infinite media does not depend much on the type of the phase function; because of the randomization of photons in semi-infinite media and for isotropic scattering $[p(\theta) = 1]$ it follows that[2,18]

$$R_{\text{ss}} = \frac{H(\mu)H(\mu_0)}{4(\mu + \mu_0)}, \tag{A3}$$

where[2]

$$H(\mu) = \exp\left[-\frac{\mu}{\pi}\int_0^\infty \ln\left(1 - \frac{\arctan \eta}{\eta}\right)\frac{d\eta}{1 + \mu^2\eta^2}\right]. \tag{A4}$$

The reflection function in this case does not depend on the azimuth. It follows from Eqs. (4) that $H(0) = 1$. The function $H(0)$ increases with the value of μ almost linearly from $H(0) = 1$ to $H(1) = 2.91$,[18] and it follows approximately that

$$H(\mu) = 1 + 2\mu. \tag{A5}$$

From Eqs. (A3) and (A5) one can obtain for the isotropic scattering

$$R(\mu, \mu_0) = \frac{1}{2} + \frac{\mu\mu_0}{\mu + \mu_0} + \frac{1}{4(\mu + \mu_0)}. \tag{A6}$$

The last term in Eq. (A6) represents the contribution of the single-scattered light [see Eq. (A2) at $p(\theta) = 1$]. Thus it follows approximately for the contribution of the multiple scattering [at $p(\theta) = 1$] that

$$R_{\text{ms}} = \frac{1}{2} + \frac{\mu\mu_0}{\mu + \mu_0}. \tag{A7}$$

For anisotropic phase functions $[p(\theta) \neq 1]$ one can assume that [Eq. (A6)]

$$R(\mu, \mu_0) = \Upsilon_0\left(\frac{1}{2} + \frac{\mu\mu_0}{\mu + \mu_0}\right) + \frac{\Upsilon_1}{4(\mu + \mu_0)}, \tag{A8}$$

where Υ_0 and Υ_1 are unknown functions of angles ϑ, ϑ_0, and ϕ. Note that for isotropic scattering it follows with a high degree of accuracy [Eq. (A6)] that $\Upsilon_0 = 1$ and $\Upsilon_1 = 1$. Our calculations show that, for phase functions of cloud media,

$$\Upsilon_0 = 1,$$
$$\Upsilon_1 = 8 - 4.5 \exp[-5(\pi - \theta)] - 5 \exp(-5|\theta^* - \theta|), \tag{A9}$$

where θ^* is the rainbow angle in radians ($\theta^* = 2.4$ for water clouds in the visible[24]).

The accuracy of Eq. (A8) for the reflection function of cloud media was investigated with the exact radiative transfer code at the following input parameters: $a_{\text{ef}} = 6\ \mu\text{m}$, $\lambda = 0.7\ \mu\text{m}$, $\tau = 900$, $\vartheta = 0, 30, 60$ deg, and $\phi = 0, 90, 180$ deg.

We found that the error of Eq. (A8) is less than 5% at nadir measurements and is in the range 5–15% in other cases studied. The largest errors arise in the vicinity of the backscattering where the glory takes place. The multiple scattering suppresses the intensity of the glory and the rainbow.

Note that the reflection of light by semi-infinite nonabsorbing clouds decreases (except for the glory region[24]) with the value of the solar angle at small observation angles ($\vartheta \leq 30°$), and it can increase with the solar angle for large observation angles. This

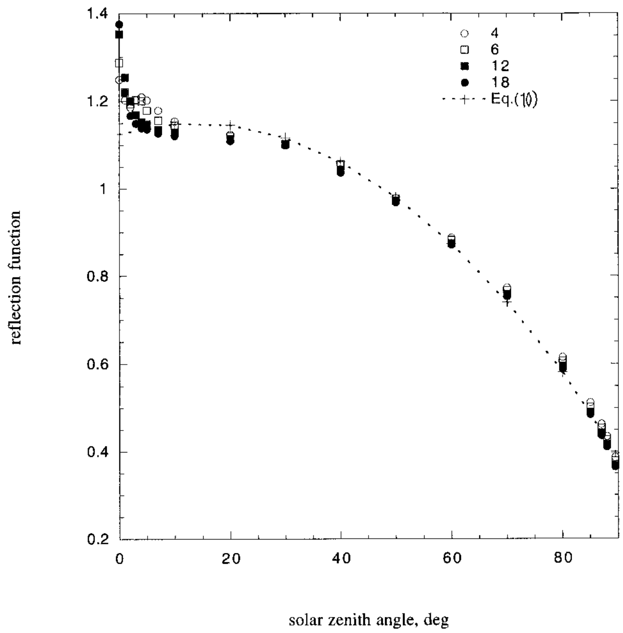

Fig. 5. Reflection function of water clouds at $\tau = 900$, $\omega_0 = 1$, $\mu = 1$, and $a_{ef} = 4, 6, 12, 18$ μm obtained with the exact radiative transfer code[8] and Eq. (A10).

feature is determined by the values of the first (multiple-scattering) and the second (single-scattering) terms in Eq. (A8). The first term decreases with the solar angle, but the second one can increase, depending on the scattering angle θ. At nadir observations, which are common for many currently operated and developed radiometers on board various satellites, the problem is simpler. In this case the reflection function depends on the value of the solar zenith angle only. This dependence for water clouds at $\tau = 900$ and $\omega_0 = 1$ is presented in Fig. 5. One can see that the reflection function practically does not depend on the water clouds' microstructure and can be approximated by the following simple equation (Fig. 5):

$$R_\infty^{\ 0}(1, \vartheta_0) = 1.125 + 0.0036\vartheta_0 - 0.00013\vartheta_0^{\ 2}, \quad \text{(A10)}$$

where the solar angle ϑ_0 is measured in degrees. Note that Eq. (A10) does not account for the glory scattering at small solar angles ϑ_0. In that case one should use the more general Eq. (A8). It is interesting that at $\vartheta_0 \in [30°, 60°]$ it follows from Eq. (A10), with an error of less than 10%, that $R(1, \vartheta_0) = 1$.

Appendix B. Symbols Used

a_{ef}	Effective radius,
C_v	volumetric concentration,
g	asymmetry parameter,
$H(\mu)$	Chandrasekhar function,
k	diffusion exponent,
$K(\mu)$	escape function,
L	geometric thickness of a layer,
$p(\theta)$	phase function,
$P(\mu)$	diffusion pattern,
r_s	ground albedo,
$R(\mu, \mu_0, \phi)$	reflection function,
$R(\mu_0)$	plane albedo,
R	spherical albedo,
$T(\mu, \mu_0, \phi)$	transmission function,
$T(\mu_0)$	transmittance,
T	global transmittance,
w	liquid-water path,
β	probability of photon absorption,
θ	scattering angle,
ϑ	observation angle,
ϑ_0	solar zenith angle,
λ	wavelength,
μ	cosine of the observation angle,
μ_0	cosine of the solar zenith angle,
ρ	density of water,
σ_{abs}	absorption coefficient,
σ_{ext}	extinction coefficient,
τ	optical thickness,
ϕ	azimuth,
χ	imaginary part of the refractive index,
ω_0	single-scattering albedo.

Appendix C. Approximate Formulas for the Radiative Characteristics of Weakly Absorbing Optically Thick Media

$$\begin{aligned}
R(\mu, \mu_0, \phi) &= R_\infty^{\ 0}(\mu, \mu_0, \phi)\exp\left[-y\,\frac{K_0(\mu)K_0(\mu_0)}{R_\infty^{\ 0}(\mu, \mu_0, \phi)}\right] \\
&\quad - TK_0(\mu_0)K_0(\mu)\exp[-(1+z)y], \\
R(\mu_0) &= \exp[-yK_0(\mu_0)] - TK_0(\mu_0)\exp[-(1+z)y], \\
R &= \sinh(yz)/\sinh[(1+z)y], \\
T(\mu, \mu_0) &= TK_0(\mu_0)K_0(\mu), \qquad T(\mu_0) = TK_0(\mu_0), \\
T &= \sinh y/\sinh[(1+z)y], \\
A &= 1 - [\sinh y + \sinh(yz)]/\sinh[(1+z)y], \\
K_0(\mu_0) &= 3/7(1 + 2\mu_0), \\
R_\infty^{\ 0}(\mu, \mu_0) &= \frac{1 + 4\mu\mu_0}{2(\mu + \mu_0)}, \\
y &= 4\left[\frac{1-\omega_0}{3(1-g)}\right]^{1/2}, \qquad z = 3/4(1-g)\tau.
\end{aligned}$$

A. Kokhanovsky held the Science Technology Agency of Japan and Alexander von Humboldt fellowships during the course of this research. His e-mail address is a.kokhanovsky@ieee.org.

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